

September 21, 2000

MATHEMATICS 110 (31)

Total Marks - 18

Quiz #1

Time: 45 minutes

Last Name: _____ Student Number: _____

[5]

1. SHORT ANSWER SECTION (Answers will be marked either RIGHT or WRONG.)

Evaluate the following EXACTLY:

a) $\cos(\pi/3) = \frac{\pi}{3} = \frac{180^\circ}{3} = 60^\circ$

$\cos 60^\circ = \cos \frac{1}{2} = \frac{1}{2}$

b) $\tan(11\pi/4) = \frac{11(180^\circ)}{4} = 495^\circ$

$495^\circ - 360^\circ = 135^\circ$
 $\tan 135^\circ = \tan(180^\circ - 45^\circ) = -\tan 45^\circ = -1$

c) $\sin(5\pi/12) = \frac{5\pi}{12} = \frac{5(180^\circ)}{12} = 75^\circ$

$\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

State whether the following statements are TRUE or FALSE:

(T) d) The relation given by $x + 2y - 6 = 0$ defines a function f with $y = f(x)$ and domain $(-\infty, \infty)$.
 $2y = 6 - x$
 $y = \frac{6-x}{2}$

(F) e) The relation given by $x + 2y^2 - 6 = 0$ defines a function f with $y = f(x)$ and domain $(-\infty, 6)$.
 $2y^2 = 6 - x$
 $y = \pm \sqrt{\frac{6-x}{2}}$

(T) f) The relation given by $x + 2\sqrt{y} - 6 = 0$ defines a function f with $y = f(x)$ and domain $(-\infty, 6)$.
 $2\sqrt{y} = 6 - x$
 $\sqrt{y} = 3 - \frac{x}{2}$
 $y = (3 - \frac{x}{2})^2$

Find the following:

g) The slope of a line through the point $(0, 1)$ and parallel to the line $y = -3x + 2$.
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{1 - 0} = -2$

h) The slope of a line through the points $(0, 1)$ and $(-1, 1)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 1}{0 - (-1)} = \frac{0}{1} = 0$
(This is a horizontal line and the slope is 0)

i) The domain of the function $f(x) = x^3 - 3x^5 + 7 - x$.

j) The domain of the function $f(x) = \sin(\sqrt{x})$.



x	y
1	-1
2	-4

[5]

2. Solve the following inequalities and express your answer using interval notation.

Show all your work.

a) $-3 \leq 5 - 2x \leq 2$

$$-3 \leq 5 - 2x$$

$$0 \leq 8 - 2x$$

$$4 \leq x$$

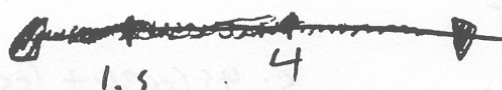
$$5 - 2x \geq 2 \quad \frac{-8 \leq -2x}{-2}$$

$$3 - 2x \geq 0 \quad 4 \geq x$$

$$x \geq 1.5$$

$$5 - 2x \leq 2 \quad \frac{-3 \leq -2x}{-2}$$

$$x \geq 1.5$$



$$[1.5, 4]$$

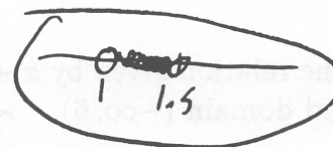
b) $\frac{x^2 + 1}{3 - 2x} < 2$

$$\frac{x^2 + 1}{3 - 2x} - 2 < 0$$

$$\frac{x^2 + 1 - 6 + 4x}{3 - 2x} < 0$$

$$\frac{x^2 + 4x - 5}{3 - 2x} < 0$$

$$\frac{(x - 5)(x + 1)}{3 - 2x} < 0$$



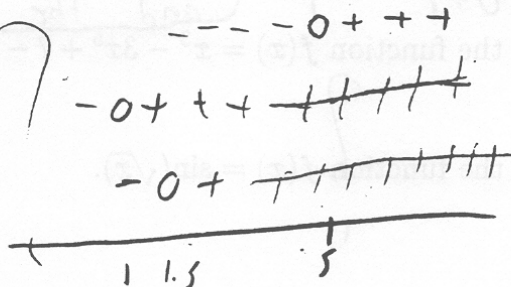
$$(-\infty, 1)$$

$$x - 5 < 0$$

$$x + 1 < 0$$

$$3 - 2x > 0$$

$$\frac{x^2 + 4x - 5}{3 - 2x} < 0$$



[4]

3. Consider the following function:

$$f(x) = 1 - |2 - x|$$

a) Give a piecewise defined formula for $f(x)$ which does not involve absolute values.

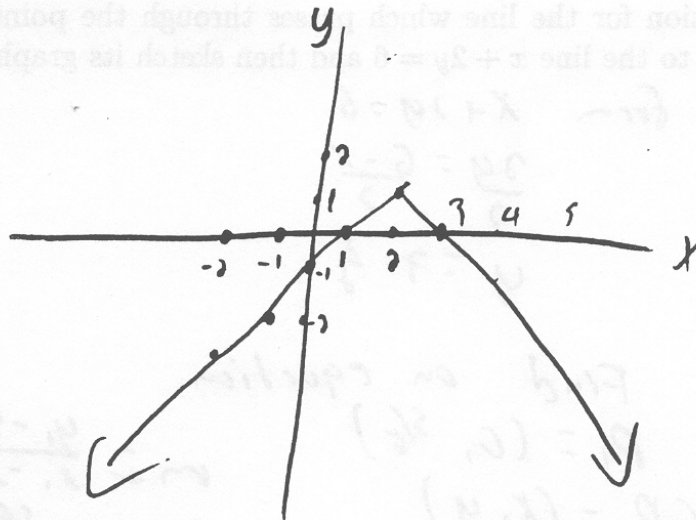
$$f(x) = \begin{cases} 2 - x & \text{if } x \geq 2 \\ -2 + x & \text{if } x < 2 \end{cases}$$

$$2 - x \quad \text{if } x \geq 2$$

$$-2 + x \quad \text{if } x < 2$$

b) Sketch the graph of $y = f(x)$.

x	y
-1	-2
0	-1
1	0
2	1
-2	-3
3	0
4	-1



c) Find the domain and range of $f(x)$.

$$D\{x \in \mathbb{R}\}$$

$$R\{y \in \mathbb{R} \mid y \leq 1\}$$

located at graph

- [2] 4. Prove or disprove that the function $f(x) = -3(x-2)^2 + 6$ is an even function.

$$\begin{array}{lcl}
 f(x) & \neq & f(-x) \\
 (x-2)^2 + 6 & & -3(-x-2)^2 + 6 \\
 (x^2 - 4x + 4) + 6 & & -3(x^2 + 4x + 4) + 6 \\
 3x^2 + 12x - 12 + 6 & & -3x^2 - 12x - 12 + 6 \\
 3x^2 + 12x - 6 & & -3x^2 - 12x - 6
 \end{array}$$

LS \neq RS

∴ this is not an even function

- [2] 5. Find an equation for the line which passes through the point $(0, 5/6)$ and is perpendicular to the line $x + 2y = 6$ and then sketch its graph.

find 2 points from $x + 2y = 6$

x	y
4	1
2	2

$$\begin{aligned}
 2y &= 6 - x \\
 y &= 3 - \frac{x}{2}
 \end{aligned}$$

Find an equation

$$P_1 = (0, 5/6)$$

$$GP = (x, y)$$

$$m = -\frac{1}{2}$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{1 - 2}{4 - 2}$$

$$m = -\frac{1}{2}$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$-\frac{1}{2} = \frac{y - 5/6}{x - 0}$$

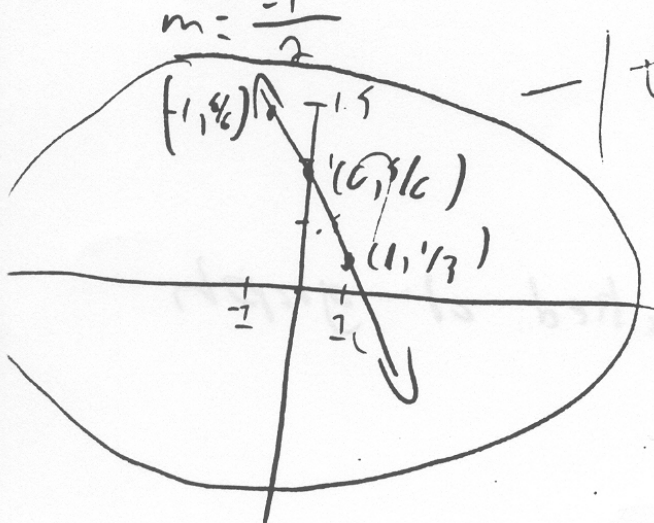
$$-x = 2y - \frac{10}{6}$$

$$\frac{10}{6} - x = 2y$$

$$\frac{5}{3} - x = 2y$$

$$\frac{5}{6} - \frac{x}{2} = y$$

x	y
1	1/3
0	5/6
-1	5/6



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October 5, 2000

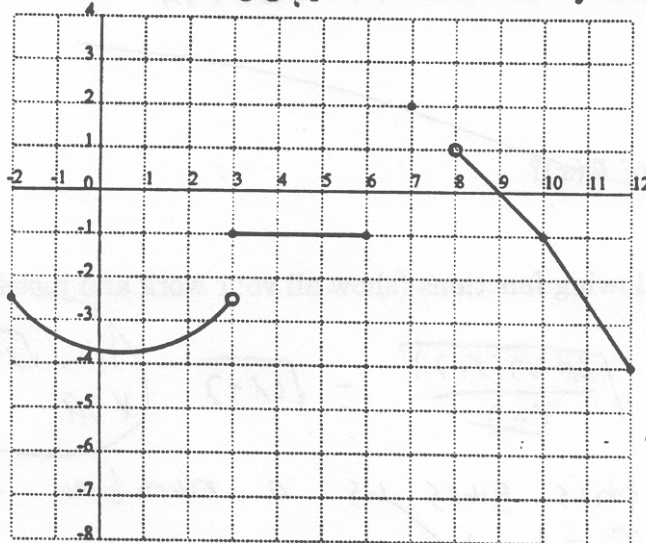
MATHEMATICS 110 (31)
Midterm #1

Total Marks - 35
Time: 80 minutes

Last Name: _____ Student Number: _____

8 [10]

1. Consider the following graph of the function f on the interval $[-2, 12]$:



- a) Assuming that the domain of f is a subset of $[-2, 12]$, give the domain and range of f ? $D_f = [-2, 6] \cup 7 \cup (8, 12]$

$$R_f = [-4, 1) \cup 2$$

- b) Give values (if possible) for the following or state that it does not exist.

$$\lim_{x \rightarrow -2^+} f(x) = -2.5$$

$$\lim_{x \rightarrow 7} f(x) = DNE$$

$$\lim_{x \rightarrow 3^-} f(x) = -2.5$$

$$\lim_{x \rightarrow 8^+} f(x) = 1$$

$$\lim_{x \rightarrow 4} (f(x) + 1)^2 = 0$$

$$\lim_{x \rightarrow 6^-} f(x) = -1$$

$$\lim_{x \rightarrow 10} f(x) = -1$$

$$f(8) = DNE$$

$$f(7) = 2$$

$$f(10) = -1$$

- c) Determine all points a in the domain of f where f is discontinuous and justify your answer by stating which continuity condition fails. State the interval(s) in the domain of f such that f is continuous on each interval and such that each interval is as large as possible.

2
pts 3, 6, 7, 8 (1) all are in domain

6, 7, 8
Because $f(x)$

[2]

$$F(x) = \sqrt{\frac{1}{e^x + 1}}$$

-2

b) What is the domain of $F(x)$?

[6]

$$\text{a) } f(x) = \sqrt{\frac{x^2 - 4}{x - 2}} = \sqrt{\frac{(x-2)(x+2)}{x-2}} = \sqrt{x+2}$$

$$\lim_{x \rightarrow 9} \sqrt{x+7} = \infty$$

- Because the $\sqrt{\quad}$ is already in the question we only take the positives.

$${}^0_c D_f = (2, \infty)$$

b) $f(x) = \frac{1}{\sqrt{6-x-x^2}}$

$$D_f = (-\infty, 2)$$

$$\lim_{x \rightarrow a} f(x)$$

$$\lim_{x \rightarrow 2^+} \frac{1}{\sqrt{6-x-x^2}} =$$

$$\lim_{x \rightarrow 4} = \frac{1}{\sqrt{6 \cdot x \cdot x}} = -\infty$$

$$\begin{array}{r|l} X & Y \\ \hline 2.1 & 3.89 \\ 1.9 & 4.29 \end{array}$$

-3

$$\begin{array}{r} 19 \\ 1.9 \\ \hline 179 \\ 140 \\ \hline 7.61 \end{array}$$

2.31
1.71

$$6-2.31 = 3.69$$

$$(-1.71) = 4.29$$

$x = 2$

[2]

4. Find all vertical asymptotes of $f(x) = \frac{x^2 + x - 6}{x^2 - 9}$ and justify your answer by finding all relevant limits.

$x^2 - 9$ and justify your answer by
 $(x+3)(x-3)$ only possibilities at 3, -

$\lim f(x)$

$$\lim_{x \rightarrow a} f(x)$$

[10]

5. Evaluate the following limits.

a) $\lim_{x \rightarrow -3^+} \frac{x^2 + 2x - 4}{(x+2)^3} = \frac{-1}{-1} = 1$

x	y
-2.9	+1.1
-2.99	+1.01
-2.999	+1.001
-2.9999	+1.0001
-3	1

b) $\lim_{x \rightarrow -2^-} \frac{x^2 + 2x - 4}{(x+2)^3} = \frac{-4}{-.001} = 4000 = \infty$

x	y
-2.1	4000
-2.01	40000
-2.001	400000

-2.0 DNE asymptote

c) $\lim_{x \rightarrow -1} \frac{2x+2}{x^2-x-2} = \frac{0}{0}$

asymptote at $x = -1$ DNE

$\frac{2(-.9)+2}{(-.9)^2-(-.9)-2} = \frac{-1.8+2}{.81+.9-2} = \frac{.2}{-1.29} = \frac{-.2}{.29}$

x	y
-1	DNE
-1.1	1
-.9	-.29

approaching diff #

No Limit or DNE

d) $\lim_{t \rightarrow 3} \frac{\sqrt{t+6}-3}{3t^2-27} = \frac{0}{0} = 0$

$81 - 27$

5. Consider the following piecewise defined function:

$$g(x) = \begin{cases} x^2 - 2 & x \leq 3 \\ x & x > 3 \end{cases}$$

a) Prove that g is discontinuous at $x = 3$.

$$\lim_{x \rightarrow a^+} g(x) = g(a)$$

$$\lim_{x \rightarrow 3^-} x^2 - 2 = 7$$

$$x \rightarrow 3^-$$

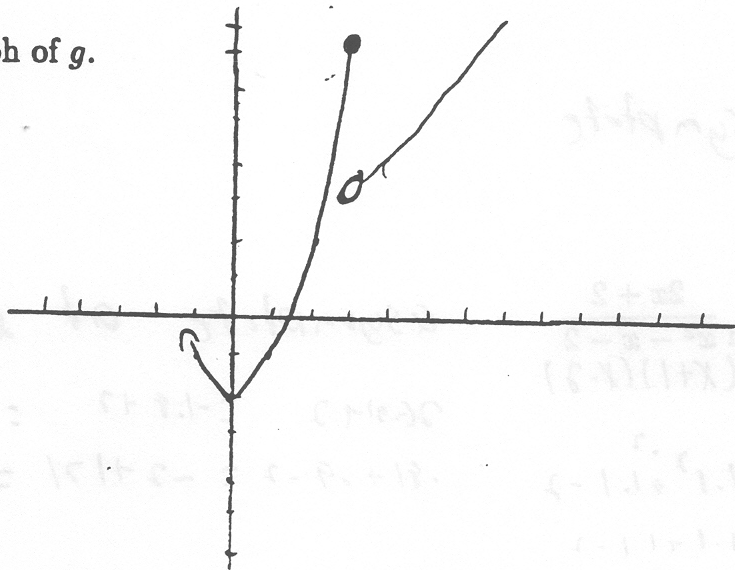
$$g(a) = g(3) = 7$$

$$\lim_{x \rightarrow 3^+} x = 3$$

3 is in domain
 $x \leq 3$

(3) $g(a) = 7 \neq 3$ so it is not continuous

b) Sketch a graph of g .



c) Consider the following piecewise defined function:

$$h(x) = \begin{cases} x^2 - 2 & x \leq 3 \\ x - k & x > 3 \end{cases}$$

For what value(s) of k is $h(x)$ continuous at $x = 3$? Justify your answer.

$$h(x) = x - k$$

$$7 = 3 - k$$

$$h(x) = 7 \leftarrow 3^2 - 2 = 7$$

$$x = 3$$

